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Upper bound on Hot Dark Matter Density from $SO(10)$ Yukawa Unification*

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Abstract

We study low-energy consequences of supersymmetric $SO(10)$ models with Yukawa unification $h_t = h_N$ and $h_b = h_\tau$. We find that it is difficult to reproduce the observed m_b/m_τ ratio when the third-generation right-handed neutrino is at an intermediate scale, especially for small $\tan\beta$. We obtain a conservative lower bound on the mass of the right-handed neutrino $M_N > 6 \times 10^{13}$ GeV for $\tan\beta < 10$. This bound translates into an upper bound on the τ-neutrino mass, and therefore on its contribution to the hot dark matter density of the present universe, $\Omega_\nu h^2 < 0.004$. Our analysis is based on the full two-loop renormalization group equations with one-loop threshold effects. However, we also point out that physics above the GUT-scale could modify the Yukawa unification condition $h_b = h_\tau$ for $\tan\beta \lesssim 10$. This might affect the prediction of m_b/m_τ and the constraint on M_N.

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Disclaimer

1. Grand unification [1] has been one of the leading guiding principles to build models of fundamental forces in nature. The introduction of a simple gauge group not only allows an elegant description of the electromagnetic, weak and strong forces, but also gives powerful insight into the structure of Yukawa couplings [2]. Supersymmetry, on the other hand, has provided a natural framework to solve the gauge hierarchy problem. On top of these theoretical ideas, there also appeared an experimental indication supporting supersymmetry combined with grand unification. Experiments as LEP [3] and SLC [4] have provided a very precise determination of the weak mixing angle. The resulting value has ruled out the minimal grand unified theories (GUTs), while showing a remarkable agreement with their supersymmetric versions based on the minimal particle content below the GUT scale M_G [5, 6]. (The absence of particle thresholds between the weak and GUT scales is often referred to as "grand desert assumption".) In the same class of theories, the partial unification of Yukawa interactions ($h_b = h_\tau$) has also proven successful [7, 8, 9], while in the non-SUSY models it has not. These facts have strengthened the motivation for studying more detailed consequences of SUSY-GUTs with the grand desert. Recent work includes more theoretical efforts on GUT model building as well as more phenomenological studies such as those on cosmic neutralino abundance, proton decay and collider signatures.

On the other hand, there are indications from astrophysics and cosmology that an intermediate scale might exist in the middle of the proposed grand desert. One indication comes from the observed deficit in the solar neutrino flux [10], which favors neutrino oscillations *à la* Mikheev–Smirnov–Wolfenstein (MSW) [11] with non-vanishing neutrino masses of $\mathcal{O}(10^{-3})$ eV [12]. If we regard the neutrino oscillation as being of ν_e–ν_μ type, then the tiny masses required for the MSW effect suggest the existence of right-handed neutrinos having an intermediate mass of 10^{10}–10^{13} GeV and acting as the source of a seesaw mechanism [13].* Another indication comes from the density fluctuations at large scales observed by COBE [14], which have set the normalization of the Harrison–Zeldovich spectrum. The result, if one assumes that dark matter is only of the cold type, is in a moderate contradiction with the observations at smaller scales [15, 16]. It has been suggested that the existence of both cold and hot dark matter components solves this problem [17]. The only particle physics candidate for hot dark matter is a light neutrino with a mass around 10 eV. If we regard this neutrino as ν_τ, then, again, a right-handed neutrino at an intermediate scale $\sim 10^{12}$ GeV is favoured. Moreover, it was also shown that scalar components of the right-handed neutrinos can play interesting cosmological roles in baryogenesis [18] and inflation [19].

Therefore, it is important to study the phenomenology of SUSY GUTs in which right-handed neutrinos are the only thresholds in the grand desert. To our knowledge, few studies of the low-energy phenomenology have been done within this context (see, *e.g.*, [21, 22]). Obviously, gauge coupling constant unification is not significantly affected because the right-handed neutrinos are gauge singlets.

This letter studies the unification of the Yukawa couplings of the third family in SUSY GUTs with right-handed neutrinos at an intermediate scale. We base our analysis on $SO(10)$ models with

*We are assuming $SO(10)$-like mass relations, where the neutrino Dirac masses are of the order of the up-type quark masses.

Yukawa unification $h_t = h_N$ and $h_b = h_\tau$. We find that an intermediate scale right-handed neutrino has a critical influence on the predicted value of m_b/m_τ. Consistency with the experimental value of m_b/m_τ implies a lower bound on the mass of the right-handed neutrino. This constraint translates into an upper bound on the τ neutrino contribution Ω_ν to the hot dark matter density in the present universe. Cosmologically interesting values of Ω_ν are strongly disfavored for small values of $\tan\beta$.[†]

2. We first describe the theoretical framework of our study. We assume a simple $SO(10)$ GUT scenario where only *small* irreducible representations (irreps) like **10**, **16**, $\overline{\mathbf{16}}$, **45**, **54** and possibly singlets are present,[‡] and where matter fields come only in **16**'s. As for the Yukawa interactions, the large value of the top Yukawa coupling suggests that it originates from a renormalizable coupling. With our assumptions this can only be $h_G \mathbf{16_3 10_H 16_3}$. (Even a **120** would not couple because of antisymmetry.) In general, the two light doublets $H_{u,d}$ of the Minimal Supersymmetric Standard Model (MSSM) contain only fractions of those in $\mathbf{10_H}$, due to mixing with doublets in other irreps. In our case these can be $\mathbf{10'_H}$'s, $\mathbf{16_H}$'s and $\overline{\mathbf{16}}_\mathbf{H}$'s, the latter ones being necessarily present in order to reduce the rank of the gauge group. The mixing strengths are in general different for H_u and H_d because $SU(2)_R$ is broken at M_G. Then the GUT boundary condition on the Yukawa couplings is $h_t = h_N = s_u h_G$, $h_b = h_\tau = s_d h_G$, where $s_{u,d}$ are the Higgs mixing angles, and in general $s_u \neq s_d$. Here the meaning of h_t, h_N, h_b, and h_τ is the obvious one. The particular case $s_u = s_d$ has been studied before [25, 24]. Actually the boundary condition

$$h_t = h_N, \quad h_b = h_\tau \tag{1}$$

is just a result of the underlying Pati-Salam $SU(4)$ symmetry [26], so that one may imagine scenarios, maybe in strings, where eq. (1) holds even without a unified simple gauge group. There are also more exotic scenarios with mixings between **16** and **10** matter fields [27], while eq. (1) keeps holding. Moreover, there is a class of models where h_b and h_τ originate solely from higher-dimension operators, while predicting the same relation [28]. We thus conclude that there is a wide class of models ($SO(10)$ and beyond) which lead to the boundary condition eq. (1).

3. In order to illustrate the low-energy consequences of b–τ Yukawa unification when a right-handed neutrino is present, we first consider a simplified picture based on one-loop renormalization group equations (RGE), without threshold corrections. This also allows for a clearer comparison

[†]There is a claim [20] that b–τ Yukawa unification combined with the value of the top quark mass recently suggested by CDF excludes the region $1.5 \leq \tan\beta \leq 30$. This claim, however, depends crucially on $\alpha_s(m_Z) \simeq 0.125$ and $m_t \lesssim 174$ GeV, for which the experimental support is not yet established. Thus, we take $\tan\beta$ as a free parameter in this letter.

[‡]Notice that *large* irreps drive the theory out of the perturbative regime; for example, if a **126** is present, the $SO(10)$ gauge coupling constant blows up below $< 8M_G$ even with the minimum particle content $\mathbf{45}+\mathbf{54}+\mathbf{126}+\overline{\mathbf{126}}+3\times\mathbf{16}$. It is also interesting to recall that, in superstring models, the above mentioned *small* irreps appear already at Kac-Moody level $k=2$, whereas **126** can appear only for $5 \leq k \leq 7$ [23]. Phenomenologically, the presence of a **126** may also give an unacceptable relation $h_\tau = 3h_b$ [24]. Finally, it is noteworthy that the absence of **126** and R-odd gauge singlets forbids right-handed neutrino masses at the renormalizable level, thus providing a dimensional argument for their being considerably smaller than M_G.

with the pure MSSM case, *i.e.* the case without right-handed neutrinos below the GUT scale. The RGE for the individual gauge and Yukawa couplings can be derived from the general formulae in Ref. [29], both at one-loop order (needed here) and at two-loop order (to be used below), and we do not write them down explicitly. Here we rather focus on the evolution of the ratio $R(\mu) \equiv h_b(\mu)/h_\tau(\mu)$ from the GUT-scale value $R(M_G) = 1$ to the low-energy value $R(m_Z)$. The latter should be compared with the experimental value of m_b/m_τ scaled up to m_Z, which we denote by $R^{exp}(m_Z)$. We first recall the situation in the pure MSSM case [30]. The RGE for R reads

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{R}{16\pi^2}\left[h_t^2 + 3(h_b^2 - h_\tau^2) - (\frac{16}{3}g_3^2 - \frac{4}{3}g_1^2)\right], \tag{2}$$

where $t \equiv \log\mu$. If all the Yukawa coupling constants are neglected, one obtains for $R(m_Z)$ the expression

$$R_g \equiv \left(\frac{\alpha_3(m_Z)}{\alpha_3(M_G)}\right)^{\frac{8}{9}} \left(\frac{\alpha_1(m_Z)}{\alpha_1(M_G)}\right)^{\frac{10}{99}}, \tag{3}$$

which is numerically larger than $R^{exp}(m_Z)$ by 30–80 %. The Yukawa contributions in eq. (2) are essential in reducing $R(m_Z)$ from its 'pure gauge' value R_g to the observed value $R^{exp}(m_Z)$. Actually, one could formally obtain arbitrarily small values for $R(m_Z)$ by taking large enough h_t at the GUT-scale [30]. In practice, initial values of $\mathcal{O}(1)$ are required to reproduce the correct $R^{exp}(m_Z)$. On the other hand, if a right-handed neutrino with mass $M_N < M_G$ is present, the MSSM eq.(2) applies only for $m_Z < \mu < M_N$. For $M_N < \mu < M_G$ the theory is the MSSM+N (in an obvious notation) and the RGE has a slight (but crucial) modification:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{R}{16\pi^2}\left[(h_t^2 - h_N^2) + 3(h_b^2 - h_\tau^2) - (\frac{16}{3}g_3^2 - \frac{4}{3}g_1^2)\right]. \tag{4}$$

The essential difference with the pure MSSM case is a manifest cancellation of *all* the Yukawa contributions, at least close to the GUT-scale, due to the boundary conditions $h_t = h_N$ and $h_b = h_\tau$. As a consequence, even if one formally allows for arbitrarily large values of the Yukawa couplings at the GUT-scale, the low-energy value $R(m_Z)$ cannot be made arbitrarily small, in contrast to the pure MSSM case. In particular, it may be impossible to reach the experimental value $R^{exp}(m_Z)$.

To make this point more explicit, it is useful to integrate formally eqs.(2) and (4), and write $R(m_Z)$ as

$$R(m_Z) = R_g \cdot e^{-Y}, \tag{5}$$

where

$$Y = \frac{1}{16\pi^2}\left[\int_{t_Z}^{t_N} h_t^2(t)\mathrm{d}t + \int_{t_N}^{t_G} (h_t^2(t) - h_N^2(t))\mathrm{d}t + 3\int_{t_Z}^{t_G} (h_b^2(t) - h_\tau^2(t))\mathrm{d}t\right], \tag{6}$$

with $t_A \equiv \log M_A$. Notice that Y should be considered as a function of the initial values of the Yukawa couplings and the mass of the right-handed neutrino. Since all the three terms in Y turn out to be positive, $R(m_Z)$ is smaller than R_g also in the present case. The question is whether or not Y can be large enough to reproduce the correct $R^{exp}(m_Z)$. Indeed the cancellation effect mentioned

above limits the size of Y, even for large initial Yukawa couplings. For a given $M_N < M_G$, Y has a (finite) upper bound $Y_{max}(M_N)$, formally obtained by taking infinite Yukawa couplings at M_G. Since $Y_{max}(M_N)$ becomes smaller for smaller values of M_N, it may be impossible to get $R(m_Z)$ as low as $R^{exp}(m_Z)$ when M_N is lighter than a certain critical value. This is why a lower bound on M_N appears. Actually, $Y_{max}(M_N)$ crucially depends on $\tan\beta$. For fixed M_N, $Y_{max}(M_N)$ is smaller when $\tan\beta$ is small, because h_b and h_τ are negligible and the last term in Y is missing. In fact this is the situation where stronger constraints on M_N will be found. On the contrary, when $\tan\beta$ is very large, the last term in Y plays a significant role (notice also the factor 3). Then, even if M_N is light, Y can be made large enough as to reduce the value of R appropriately.

The above qualitative discussion is supported not only by a numerical analysis, but also by an analytical argument which we sketch below. The argument establishes the existence of an upper bound on Y as a function of M_N independent of the initial conditions on the Yukawa couplings. Even if one cannot solve exactly the coupled system of RGE, exact inequalities can be found that lead to the final result. We consider here the most interesting case where h_b and h_τ are negligible. Then we can write $Y = Y_1 + Y_2$, where Y_1 and Y_2 are the first and second terms in eq. (6). We denote by $h_0(t)$ the solution of the RGE for $h_t(t)$ in the pure MSSM case with boundary condition $h_0(M_G) \to \infty$. Its explicit expression is [30] $h_0^2(t) = 4\pi^2(-F'(t)/3F(t))$, where $F(t) \equiv \int_t^{t_G} \mathrm{d}t' \left(\alpha_3(M_G)/\alpha_3(t')\right)^{-\frac{16}{9}} \left(\alpha_2(M_G)/\alpha_2(t')\right)^3 \left(\alpha_1(M_G)/\alpha_1(t')\right)^{\frac{13}{99}}$. A study on the RGE of $h_t(t)$ for $t < M_N$ and $t > M_N$, with a given boundary condition $h_t(M_G)$, leads easily to the absolute upper bound $h_t(t) < h_0(t)$, valid for any t and independent of $h_t(M_G)$. This gives immediately an upper bound on Y_1,

$$Y_1 < Y_1^*(M_N) \equiv \frac{1}{12} \log \frac{F(m_Z)}{F(M_N)} \tag{7}$$

where we have replaced for convenience the arguments t_A with M_A. A simple bound on Y_2 can be derived *e.g.* by studying the RGE for the ratio $\tilde{R}(t) \equiv h_t(t)/h_N(t)$, which reads

$$\frac{1}{\tilde{R}} \frac{\mathrm{d}\tilde{R}}{\mathrm{d}t} = \frac{1}{16\pi^2} \left[3(h_t^2 - h_N^2) - (\frac{16}{3} g_3^2 + \frac{4}{15} g_1^2) \right]. \tag{8}$$

If one formally integrates the above equation term by term between t_N and t_G, the right hand side contains Y_2 and easily integrable gauge terms. The left hand side gives just $-\log \tilde{R}(t_N)$, which is negative (one can easily prove that $\tilde{R}(t) > 1$ from a different form of eq.(8)). Therefore one obtains an absolute bound

$$Y_2 < Y_2^*(M_N) \equiv \frac{8}{27} \log \frac{\alpha_3(M_N)}{\alpha_3(M_G)} - \frac{2}{297} \log \frac{\alpha_1(M_N)}{\alpha_1(M_G)}, \tag{9}$$

Notice that the bounds (7) and (9) are independent of the initial values $h_t(M_G) = h_N(M_G)$, so they hold even in the formal limit $h_t(M_G) \to \infty$. In conclusion, for a given $M_N < M_G$ we obtain explicitly an absolute upper bound $Y^*_{max}(M_N)$ on Y which is a monotonically increasing function of M_N [§]

$$Y \leq Y_{max}(M_N) < Y^*_{max}(M_N) = Y_1^*(M_N) + Y_2^*(M_N). \tag{10}$$

[§] Although $Y_{max}(M_N)$ is always monotonically increasing, the same applies to the analytic bound $Y^*_{max}(M_N)$ for

This in turn gives an absolute lower bound on $R(m_Z)$

$$R(m_Z) > R_g \cdot e^{-Y^*_{max}(M_N)} = R_g \cdot \left(\frac{F(m_Z)}{F(M_N)}\right)^{-\frac{1}{12}} \left(\frac{\alpha_3(M_N)}{\alpha_3(M_G)}\right)^{-\frac{8}{27}} \left(\frac{\alpha_1(M_N)}{\alpha_1(M_G)}\right)^{\frac{2}{297}}, \tag{11}$$

proving that one cannot make $R(m_Z)$ arbitrarily small even in the formal limit $h_t(M_G) \to \infty$. This bound can be compared with $R^{exp}(m_Z)$, so that a lower bound on M_N can be inferred.

4. We have shown an analytic bound on R in which h_N was allowed to go to infinity at M_G. In practice h_N will not be taken larger than $\mathcal{O}(1)$. Then the effect of a right handed neutrino with mass around 10^{13} GeV can be roughly estimated by treating it as a threshold correction to R at M_G [24]. This gives $R \simeq 1 + \frac{h_N^2}{16\pi^2} \log(M_G/M_N)$, which amounts to an $\mathcal{O}(10\%)$ increase in R. This 10% is a critical amount when comparing to the experimental value, since already in the pure MSSM case the predicted R lies in the upper part of the allowed range. Nonetheless it is clear that all other effects of the same order may have a huge impact on the bound on M_N. This is because the correction to R from the right-handed neutrino depends roughly logarithmically on M_N. Then, in order to make our analysis more accurate, we have to discuss SUSY and GUT thresholds and use two-loop RGE. In fact the effects of these, and especially the first two, can possibly pile up to $\mathcal{O}(10\%)$. We also attempt an estimate of the possible effects from physics above M_G, even though we are aware that our understanding of these can only be qualitative. Finally, by varying the QCD coupling $\alpha_s(m_Z)$ within its experimental range we get an $\mathcal{O}(10–20\%)$ effect, which we treat with the proper attention. Our reference value of $\alpha_s(m_Z)$ is the one from the Z-boson hadronic width $\alpha_s(m_Z) = 0.124 \pm 0.007$ [31], and we allow a 2 σ range.

To obtain quantitative constraints on the mass of the right-handed neutrino, we perform a numerical analysis based on two-loop RGE with one-loop threshold corrections. In order to be able to discuss the dependence on $\alpha_s(m_Z)$ [8, 24], we take the following procedure. We *define* M_G as the scale where α_1 and α_2 meet, and take $h_t = h_N$, $h_b = h_\tau$ at the same scale.¶ On the other hand, we take $\alpha_s(m_Z)$ as an independent parameter, and allow $\alpha_3 \neq \alpha_1 = \alpha_2$ at M_G. This difference could be accounted for by GUT threshold effects. The gauge coupling constants in the MSSM at m_Z are $\alpha_1^{-1}(m_Z) = 59.1 + \frac{1}{2\pi}\left(\frac{12}{5}\log\frac{m_{SUSY}}{m_Z} + \frac{1}{10}\log\frac{m_A}{m_Z}\right)$, $\alpha_2^{-1}(m_Z) = 29.4 + \frac{1}{2\pi}(4\log\frac{m_{SUSY}}{m_Z} + \frac{1}{6}\log\frac{m_A}{m_Z})$, and $\alpha_3^{-1}(m_Z) = \alpha_s^{-1}(m_Z) + \frac{4}{2\pi}\log\frac{m_{SUSY}}{m_Z}$. Here the logarithms take care of the matching between the SM and the MSSM; we take the additional Higgs doublet at m_A and the rest of the SUSY particles at m_{SUSY}. (The non-logarithmic parts and the translation from $\overline{\text{MS}}$ to $\overline{\text{DR}}$ are numerically small and have been neglected.) We also match the couplings in the MSSM+N to those in the MSSM at $\mu = M_N$,‖

$$\begin{array}{ll} h_b(M_N)|_{\text{MSSM}} = h_b(M_N)|_{\text{MSSM+N}}\,, & h_\tau(M_N)|_{\text{MSSM}} = h_\tau(M_N)|_{\text{MSSM+N}}\,(1-\varepsilon), \\ h_t(M_N)|_{\text{MSSM}} = h_t(M_N)|_{\text{MSSM+N}}\,(1-\varepsilon), & h_N(M_N)|_{\text{MSSM}} = h_N(M_N)|_{\text{MSSM+N}}\,(1-2\varepsilon), \end{array} \tag{12}$$

$M_N \gtrsim 10^{10}$ GeV only. However, this is just the region of interest. We add that stronger analytic bounds than the simple one described here can be found.

¶Numerically, M_G turns out to be around $2 \cdot 10^{16}$ GeV. GUT threshold corrections on the Yukawa unification conditions will be discussed later.

‖We define the right-handed neutrino mass M_N by $M_N = \overline{M}_N(M_N)$, where $\overline{M}_N(\mu)$ is the $\overline{\text{DR}}$ running mass.

where $\varepsilon = \frac{h_N^2(M_N)}{32\pi^2}$. $h_N(\mu)$ in the MSSM is defined in such a way that the coefficient of the dimension-five operator $(LH_u)^2$ is $h_N^2(\mu)/M_N$.

For any fixed values of (h_t, h_b) at M_G, the actual calculation is done as follows. We use the two-loop RGE of the MSSM and the MSSM+N. By using an iterative procedure, we find the values of M_G, $\alpha_1(M_G)$ and $\alpha_3(M_G)$ which reproduce the values of $\alpha_1(m_Z)$, $\alpha_2(m_Z)$ and $\alpha_3(m_Z)$ defined above. At the same time we obtain the values of the Yukawa couplings at m_Z. The resulting b–τ Yukawa ratio $R(m_Z)|_{\text{MSSM}}$ is then matched to the ratio of the corresponding masses in the broken electroweak theory $R(m_Z) = m_b(m_Z)/m_\tau(m_Z)$,

$$R(m_Z) = (1 + k_b - k_\tau + f_R)\; R(m_Z)|_{\text{MSSM}}\,, \tag{13}$$

where k_b, k_τ are the threshold corrections due to the SUSY particles, while f_R is that from the additional Higgs doublet in the MSSM. The exact expressions are found in Ref. [24]. In the following we will only focus on the logarithmic terms in $k_{b,\tau}$. We will discuss the potentially large non-logarithmic ones (k_b' in Ref. [24]) at the end of the letter. Note that also $R(m_Z)|_{\text{MSSM}}$ depends implicitly on m_{SUSY} and m_A due to the threshold dependence of the MSSM gauge coupling constants $\alpha_i^{-1}(m_Z)$ (see above). For instance, in the small $\tan\beta$ region, the dependence of R on the SUSY particle masses turns out to be approximately**

$$\frac{\delta R}{R} \simeq \frac{1}{2\pi}\left(-0.1\,\log\frac{m_{SUSY}}{m_Z} + 0.1\,\log\frac{m_A}{m_Z}\right). \tag{14}$$

The threshold corrections roughly cancel when $m_{SUSY} = m_A$, which is consistent with the results in Ref. [8]. In the final step, the predicted $R(m_Z)$ is compared to $R^{exp}(m_Z)$. Notice that also $R^{exp}(m_Z)$ has an important dependence on $\alpha_s(m_Z)$, since it is obtained by scaling the b–τ mass ratio up to m_Z. GUT-scale threshold corrections could also affect the boundary condition in eq. (1). Though we neglect such corrections in the following numerical analysis, we will discuss their possible effects later.

We first illustrate the dependence of R on $h_t(M_G)$ for the region $\tan\beta \lesssim 10$, where both h_b, h_τ get renormalized homogeneously and the result depends very weakly on $\tan\beta$. We show curves for different values of M_N in Fig. 1, taking $m_{SUSY} = m_A = m_Z$, $h_b = h_\tau = 0.01$ at M_G, and $\alpha_s(m_Z) = 0.11$ (Fig. 1a) or 0.12 (Fig. 1b). For comparison, we also show the values of $R^{exp}(m_Z)$ corresponding to the $\overline{\text{MS}}$ mass $m_b(m_b) = 3.9$, 4.15, and 4.4 GeV. This is the range which is obtained from QCD sum rules [32, 24].†† First of all, it is clear from the figures that it becomes harder to reconcile $R(m_Z)$ with $R^{exp}(m_Z)$ for larger $\alpha_s(m_Z)$ and lower M_N. For $\alpha_s(m_Z) = 0.12$, one needs

** In a more general case where the SUSY spectrum is nondegenerate, the dependence of R on the SUSY particle masses can again be summarized by an effective m_{SUSY}. It turns out that the dependence of R on colorless particle masses is very weak, and m_{SUSY} can be interpreted as a geometric average of squark and gluino masses. After considering generic splittings in the SUSY spectrum, we found that the value of R varied only between two extreme cases, $m_{SUSY} = 1$ TeV, $m_A = m_Z$, which gives the smallest R, and $m_{SUSY} = m_Z$, $m_A = 1$ TeV which gives the largest R. We stress that the approximate formula eq. (14) cannot be applied for $m_A \lesssim m_Z$.

††The uncertainty on $m_b(m_b)$ is dominated by our lack of knowledge on the $\mathcal{O}(\alpha_s^2)$ corrections to QCD sum rules. See Ref. [24].

very large $h_t(M_G) \gtrsim 3$ even for $M_N = M_G$.[‡‡] Moreover, even if increasing $h_t(M_G)$ makes $R(m_Z)$ decrease, the suppression effect becomes weaker as we lower M_N. Note also that the value of $R(m_Z)$ depends only weakly on $h_t(M_G)$ for $h_t(M_G) \gtrsim 2$, especially for $M_N < 10^{14}$ GeV. Such a 'fixed point' behaviour and the dependence on M_N were expected on the basis of the analytic discussion above (see e.g. eqs.(10-11)). As we lower M_N, we can check whether the curves reach the region of $R^{exp}(m_Z)$ and then infer a lower bound on M_N. In the following, when scanning the space of $(h_t(M_G), h_b(M_G))$, we will only take for definiteness $h_{t,b}(M_G) < 2$. This reference value is motivated both by the above observation and by perturbativity reasons (see also point 2 below).

Next we show lower bounds on M_N as functions of $\tan\beta$ in Fig. 2. In order to obtain conservative bounds we take the maximum value of $m_b(m_b) = 4.4$ GeV. For most of the $\tan\beta$ values, we obtain stringent lower bounds on M_N. We show the cases of two representative SUSY particle spectra (a) $m_{SUSY} = 1$ TeV (conservative) and (b) $m_{SUSY} = m_Z$ (more stringent), while always keeping $m_A = m_Z$ (conservative). Curves are shown for three values of $\alpha_s(m_Z) = 0.110$ (solid), 0.117 (dotted), and 0.125 (dashed), where the lower ones correspond to case (a) and the upper ones to (b). The values of M_N *above* the curves are consistent with b–τ Yukawa unification. The constraint becomes stronger for larger values of $\alpha_s(m_Z)$, smaller values of m_{SUSY}, and larger values of m_A. The curves do not extend to the region $\tan\beta \gtrsim 58$ because of the constraint $h_b = h_\tau < 2$ at M_G. When the curves reach M_G, it means that the lower values of $\tan\beta$ are not consistent with b–τ Yukawa unification even when $M_N = M_G$ (with $h_t(M_G) < 2$). Possible effects from the non-logarithmic SUSY threshold corrections and physics above the GUT-scale will be discussed in points (1) and (2) below.

The lower bound on M_N has a very interesting implication for the value of Ω_ν, the ν_τ contribution to the hot dark matter density of the present universe. The mass of ν_τ is related to M_N and h_N via the seesaw formula,[*]

$$m_{\nu_\tau} = \frac{h_N^2 v^2 \sin^2\beta}{M_N}, \tag{15}$$

where $v = 174$ GeV and h_N in the effective theory below M_N was defined after eq.(12). On the other hand, the cosmic energy density of a light neutrino of mass $m_\nu \lesssim 1$ MeV is simply proportional to m_ν [33],

$$\Omega_\nu h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}}. \tag{16}$$

Here, h is the normalized Hubble constant, $h = H_0/(100 \text{ km sec}^{-1}\text{ Mpc}^{-1})$. Note that $h_N(m_Z)$ cannot be larger than an infrared fixed point value, which is $h_N(m_Z) \leq 0.8$ for the cosmologically interesting region $M_N \lesssim 10^{13}$ GeV. Using that value in eqs. (15-16), we obtain the maximum possible value of $\Omega_\nu h^2$ for a given M_N. By requiring that ν_τ gives a certain contribution to Ω, one

[‡‡]For $\alpha_s(m_Z) \gtrsim 0.12$ and $h_t(M_G) < 2$, the predicted value of $R(m_Z)$ is larger than the experimental upper bound. Consistency could be possibly restored by allowing $\mathcal{O}(5\%)$ GUT- or SUSY-scale threshold corrections. See discussion below eq. (19).

[*]Here we neglect SUSY- and weak-scale threshold corrections on m_{ν_τ}, which are at most $\mathcal{O}(5\%)$. Neglecting them is also justified because they affect the constraint on M_N only linearly. In contrast, the analogous corrections to $R(m_Z)$ discussed above are larger and affect the constraint on M_N exponentially.

can put an *upper bound* on M_N. We indicated such bounds for $\Omega_\nu h^2 = 0.01$, 0.1, and 1 in Fig. 2 with dotted lines. We find that the cosmologically interesting range $\Omega_\nu h^2 \gtrsim 0.1$ survives only for large $\tan\beta$, especially for larger values of $\alpha_s(m_Z)$. For $\alpha_s(m_Z) \gtrsim 0.125$, we find solutions only for very large $\tan\beta$, and they completely disappear for $\alpha_s(m_Z) > 0.13$. In other words, this analysis puts an upper bound on the cosmic hot dark matter density as a function of $\tan\beta$. For instance, we obtain

$$\Omega_\nu h^2 < 0.004, \tag{17}$$

for $\tan\beta < 10$ even with the most conservative parameters $\alpha_s(m_Z) = 0.11$ and case (a).

5. We now discuss the GUT-scale threshold corrections on $h_t = h_N$, $h_b = h_\tau$. They mainly come from the mass splittings within the heavy gauge multiplet as far as there are no other sizable Yukawa couplings of $\mathbf{16}_3$ to other GUT-scale fields [24]. For instance, mass splittings of $\mathcal{O}(10)$ modify the boundary condition by less than 5 %.† A 5 % correction on h_t/h_N leads to less than 1 % change in the prediction of $R(m_Z)$ so that it will not change the constraint at all. However a 5 % threshold correction on h_b/h_τ results in a 5 % correction in $R(m_Z)$, leading to a two order of magnitude change on M_N (see Fig. 1) and one might imagine a situation where the combined effects of $\alpha_s(m_Z) = 0.11$, SUSY and GUT thresholds pile up to allow $M_N = 10^{12}$ GeV, even at small $\tan\beta$. Situations like this are however rather extreme. When $\alpha_s(m_Z) \simeq 0.12$ (close to the centre of its range) and SUSY particles are light (as they should be), then GUT threshold corrections must amount to 15% to make $M_N = 10^{12}$ GeV possible, which requires $\mathcal{O}(1000)$ mass splittings in the gauge multiplet. Relaxation of the maximum allowed $h_t(M_G)$ up to 3.3 [8] could also weaken the constraint on $\Omega_\nu h^2$, but no more than by a factor of 7.

Our bound on Ω_ν has been derived in the approximation in which family mixings are neglected. Then one might wonder whether, or under what conditions, the mixings can sizably affect the bound. In order to evade our bounds, the full mass matrices should satisfy the following two requirements: i) The interaction eigenstate $N_3 \subset \mathbf{16_3}$ with Yukawa coupling $h_N = h_t$ has an $\mathcal{O}(1)$ overlap with a mass eigenstate of mass $\sim M_G$. Then the effective h_N appearing in the RGE below M_G is smaller than h_t, and the effect of h_t in the running of R is no longer "exactly" compensated. ii) There still is a left-handed neutrino which contributes to a sizable portion of Ω. In order to study the consequences of these two requirements, we consider the case of two families, which may represent the second and third ones. The superpotential reads as $W = \hat{h}_e^{ij} L_i e_j H_d + \hat{h}_N^{ij} L_i N_j H_u + \frac{1}{2}\hat{M}_{ij} N_i N_j$ with $i, j = 2, 3$, where L, e and N are, respectively, lepton doublet, right-handed charged lepton, and right-handed neutrino superfields. The smallness of CKM angles and $SO(10)$ relations among Yukawa matrices suggest that both $\hat{h}_e^{ij}$ and $\hat{h}_N^{ij}$ are hierarchical in the same basis. By going to the basis where $\hat{h}_N$ is diagonal, we parametrize

$$\hat{h}_N = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix} = \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix} h_N, \tag{18}$$

†As explained above, our procedure implicitly requires a GUT-scale threshold correction on $\alpha_3(M_G)$ to reproduce $\alpha_s(m_Z)$; however, the required correction amounts to at most 5 % if we vary $\alpha_s(m_Z) = 0.11$–0.14 and $m_{SUSY} = m_Z$–1 TeV. Though the corrections on α_3 and h_b/h_τ are not directly correlated, this value gives us a rough idea of the magnitude of the GUT-scale threshold corrections.

$$\hat{M} \;=\; \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{33} \end{pmatrix}, \tag{19}$$

where $M_{ij} \lesssim \mathcal{O}(M_G)$ and we expect $\epsilon \sim m_c/m_t \sim 10^{-2}$. Requirement i) implies that at least one of the M_{ij} should be $\mathcal{O}(M_G)$, but not M_{22} alone. The light neutrino mass matrix can be written as

$$\mathcal{M}_\nu = \langle H_u \rangle^2 \left(\hat{h}_N \hat{M}^{-1} \hat{h}_N^{\mathrm{T}} \right) = \frac{\langle H_u \rangle^2 h_N^2}{\mathrm{Det}(\hat{M})} \begin{pmatrix} \epsilon^2 M_{33} & -\epsilon M_{23} \\ -\epsilon M_{23} & M_{22} \end{pmatrix}. \tag{20}$$

Requirement ii) is expressed as $(m_1^2 + m_2^2)^{1/2} \geq v^2 \sin^2\beta/(\delta \cdot M_G)$, where $m_{1,2}$ are the eigenvalues of $\mathcal{M}_\nu$, and we need $\delta \lesssim 10^{-3}$ for a neutrino mass in the eV range. Therefore one needs

$$|\mathrm{Det}\hat{M}| \lesssim \delta \left(\epsilon^4 M_{33}^2 + 2\epsilon^2 M_{23}^2 + M_{22}^2 \right)^{1/2} M_G, \tag{21}$$

where we used $h_N \sim 1$. The above inequality implies a hierarchy in the eigenvalues of $\hat{M}$. This leads to strong constraints on $\hat{M}$. If all the entries of $\hat{M}$ are $\mathcal{O}(M_G)$, we need a fine-tuning of $\mathcal{O}(\delta)$ to obtain the small determinant in eq. (21).* We can avoid such a fine-tuning only if $\hat{M}$ is hierarchical in the same basis where $\hat{h}$ is diagonal with $M_{22} \lesssim \delta\epsilon^2 M_G$, $M_{23} \lesssim \delta^{1/2}\epsilon M_G$ and $M_{33} \sim M_G$. In this case $\hat{M}$ could be viewed as a simple consequence of an abelian horizontal symmetry, but notice that the expansion parameter $\delta^{1/2}\epsilon$ would be rather small. Moreover, the hot dark matter candidate would be predominantly ν_μ, and the MSW oscillation should occur between ν_e and ν_τ. This is a new possibility which may be worth of further study. Nonetheless both the latter possibility and the fine-tuned case mentioned above show rather extreme features, thus enhancing the importance of the cases to which our analysis correctly applies. These cases include the reasonable situation in which there is no hierarchy among right handed neutrino masses, and they all decouple from the theory essentially at a single scale.

6. Finally we point out two generic uncertainties in b–τ Yukawa unification which might affect the predictivity on R. These exist even in the pure MSSM case. Of course, such uncertainties may also affect the bound on M_N.

(1) SUSY-scale non-logarithmic threshold corrections to R for large $\tan\beta$ [24, 34]. Typically the largest corrections appear in the b mass via two diagrams, one involving gluino propagation and the other involving higgsinos. The resulting correction to m_b can be written as

$$\frac{\delta m_b}{m_b^{\mathrm{tree}}} = \tan\beta \left(\frac{2\alpha_s}{3\pi} \frac{\mu m_{\tilde{g}}}{m_1^2} + \frac{h_t^2}{16\pi^2} \frac{\mu A_t}{m_2^2} \right), \tag{22}$$

where $m_{\tilde{g}}$ and A_t are respectively the gluino mass and the stop trilinear coupling, while m_1 and m_2 represent effective SUSY masses out of the loop integrals. In particular m_1 roughly

*The tuning implies that that the leading contribution to $\hat{M}$ is rotated by $\mathcal{O}(1)$, with respect to the leading one in $\hat{h}_{e,N}$. It may be interesting to ask whether this can result from a flavour symmetry.

corresponds to the maximum between the sbottom and gluino masses, while m_2 represents that between stop and higgsino (more exact expressions are found in [24]). When the SUSY parameters are all of the same order of magnitude, the typical size of the expression inside brackets in eq. (22) is $\sim 1\%$. Then for $\tan\beta = \mathcal{O}(1)$ we can safely neglect them. On the other hand for $\tan\beta \sim 10$ their effect could be a 10% reduction of R (notice that the sign of eq. (22) is not fixed). As already stated, a 10% reduction in R is critical for the bounds on M_N, so that $\Omega_\nu \sim 0.1$ could be allowed for $\tan\beta > 10$. Notice however that δm_b depends rather strongly on the features of the SUSY spectrum and becomes negligible when either μ or the gluino mass (and A_t) are somewhat smaller than the squark masses. Only after an experimental determination of the SUSY mass parameters, will we be able to make conclusive statements on the $\tan\beta \gtrsim 10$ region. It remains however true that in a relevant region of parameter space (with small δm_b), the bounds shown in the figure are valid.[†]

(2) Corrections from physics above the GUT-scale. In our analysis we allowed a relatively large $h_t(M_G) < 2$, which implies that the Yukawa Landau pole, and presumably some new physics, can be as close as $8M_G$.[‡] In general, the presence of new physics at a scale $\Lambda > M_G$ induces non-renormalizable operators in the effective theory at the GUT-scale. Such operators will in general affect eq. (1) with contributions scaling as powers of $M_G/\Lambda \equiv \eta$, and one could expect $\eta \sim 0.1$ (*e.g.* from the above observation). Indeed, the order of magnitude of this ratio naturally suggests that flavour mixings and mass ratios are generated by the same type of effects. From this viewpoint, it is legitimate to make the order of magnitude interpretation $\eta \sim V_{cb} \simeq 0.05$. Flavour scenarios of this type have been depicted in Ref. [38, 28]. We will therefore use the indicative value above to estimate the effects of non-renormalizable operators, which we classify below.

A first class of operators is obtained by inserting $SO(10)$ breaking Higgs fields in the renormalizable term $\mathbf{16_3 10_H 16_3}$. Our assumption that there be *small* irreps only ($\mathbf{45}$'s, $\mathbf{54}$'s, $\mathbf{16}$'s, $\overline{\mathbf{16}}$'s and $\mathbf{10}$'s) constrains the lowest possible correction to be $\mathcal{O}(\eta^2) \sim 1\%$. These might come from $\mathbf{16_3 10_H}\langle\mathbf{45}\rangle\langle\mathbf{45}\rangle\mathbf{16_3}/\Lambda^2$ and $\mathbf{16_3 10_H}\langle\mathbf{16_H}\rangle\langle\mathbf{16_H}\rangle\mathbf{16_3}/\Lambda^2$.[§] Such effects can be safely neglected.

A second class of operators involves the other Higgs fields $\mathbf{10'_H}$, $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$, rather than the $\mathbf{10_H}$ appearing in the renormalizable term. The only possible dimension five operators are $\mathbf{16_3 10'_H}\langle\mathbf{54}\rangle\mathbf{16_3}$, $\mathbf{16_3 10'_H}\langle\mathbf{45}\rangle\mathbf{16_3}$, $\mathbf{16_3 16_H}\langle\mathbf{16_H}\rangle\mathbf{16_3}$ and $\mathbf{16_3}\overline{\mathbf{16}}_\mathbf{H}\langle\overline{\mathbf{16}}_\mathbf{H}\rangle\mathbf{16_3}$. None of them

[†] It must be added that the approximate symmetries which render large $\tan\beta$ natural suppress δm_b [24]. On the other hand, these approximate symmetries require $m^2_{SUSY} \gtrsim \tan\beta m_Z^2$ because of the LEP constraint $\mu, m_{1/2} \gtrsim m_Z$. Therefore the case $\mu m_{\tilde g}/m^2_{SUSY} \sim 1$ and the case $\mu m_{\tilde g}/m^2_{SUSY} \sim 1/\tan\beta$ require a comparable fine-tuning in the Higgs sector [35, 36]. However, phenomenological constraints from the observed rate for $b \to s\gamma$ again seem to favor the small δm_b case [36, 37].

[‡] If one requires perturbativity up to $M_{Planck}/\sqrt{8\pi}$, one needs $h_t(M_G) \leq 1.5$. On the other hand, notice that the value $h_t(M_G) = 3.3$ allowed in previous studies [8] implies that the Landau pole is closer than $2M_G$.

[§] Note that $\langle\mathbf{54}\rangle$ preserves $O(6)\times O(4)$ (Pati-Salam group), and cannot modify eq. (1). Also, $\mathbf{16_3 10_H}\langle\mathbf{45}\rangle\mathbf{16_3}$ respects eq. (1) due to the symmetry between two $\mathbf{16_3}$.

affects $h_b = h_\tau$, while the last one may induce a correction $\mathcal{O}(\eta) \sim 10\%$ to $h_t = h_N$. Such a correction does not affect our analysis, as we already stated (see discussion below eq. (17)). On the other hand, dimension six operators can affect $h_b = h_\tau$. Actually, the relevance of this class of operators strongly depends on the magnitude of h_b, h_τ themselves, *i.e.* on $\tan\beta$. When $h_{b,\tau} \gtrsim \eta$ (or $\tan\beta \gtrsim \eta m_t/m_b \sim 10$), dimension six terms can give corrections which are $\mathcal{O}(\eta^2/h_{b,\tau}) \lesssim \eta \sim$ 5–10%. We recall that, for $h_{b,\tau} \ll 1$, H_d sits mainly in other Higgs fields than $\mathbf{10_H}$. In fact, for $h_{b,\tau} \sim \eta$, the above dimension five operators become a natural *source* for these Yukawa couplings. When $h_{b,\tau} \lesssim \eta$, *i.e.* $\tan\beta \lesssim 10$, the dimension five operators themselves have to be suppressed, since they would typically yield $\mathcal{O}(\eta)$ Yukawa couplings. This could result from a flavor symmetry. In this situation the dimension six operators become relevant and can finally even dominate $h_{b,\tau}$. Since these terms can give both $h_b = h_\tau$ and $3h_b = -h_\tau$ (via "composite" **126** combinations), or a mixture of both, the only way not to lose predictivity completely is to suppose that only the first type of terms exists. With this assumption the corrections to $h_b = h_\tau$ are again expected to be at most $\mathcal{O}(\eta)$, and could be actually $\mathcal{O}(\eta^n)$ (with $n > 1$) in more specific flavour models.

In short, $h_b = h_\tau$ is an automatic consequence of gauge symmetry and field content alone for $h_{b,\tau} \gtrsim \eta$. For smaller $h_{b,\tau}$, the relation $h_b = h_\tau$ should probably result from flavor symmetries.

We are aware that the last class of corrections is less under our control than all the other ones discussed so far. In fact, an opposite viewpoint would emerge if neutrino-oscillation experiments like CHORUS or NOMAD should find $m_{\nu_\tau} \gtrsim$ eV. This discovery could indeed be used to probe the existence of higher dimensional effects. For instance, should the observations suggest $M_N = 10^{12}$ GeV, then we would need $R(M_G) = 1.05$, 1.13, 1.23, for $\alpha_s(m_Z) = 0.110$, 0.117, 0.125, respectively. It is manifest that for the larger values of α_s we need the second class of effects (and larger than expected), while for the smaller α_s GUT thresholds alone could account for the deviation of $R(M_G)$ from unity. These remarks could be a useful guide in building realistic GUT models.

Therefore, there are possible corrections to the prediction of $R(m_Z)$ for both $\tan\beta \gtrsim 10$ and $\tan\beta \lesssim 10$ from very different origins (indeed the reason is the same; namely $m_b \ll m_t$). For large $\tan\beta$, non-logarithmic SUSY threshold effects can modify the prediction, but they can be calculated after the SUSY spectrum will be known. For small $\tan\beta$, corrections arising from physics beyond the GUT-scale could lead to two different classes of effects. When only first class effects are present, like in Ref. [38], then the condition in eq. (1) is robust and so are our bounds on M_N. On the other hand, when $\tan\beta \ll m_t/m_b$, reasonable flavor physics scenarios with the second class of operators may generically lead to a $\mathcal{O}$(5–10%) shift in the boundary condition, which is typically non-negligible for our purposes. However, in specific models this effect may also be further suppressed by symmetries. Moreover, even with $\mathcal{O}$(5–10%) corrections, there still are stringent limits on M_N for $\alpha_s(m_Z) \gtrsim 0.12$. Note also the new physics scale Λ could be larger $\gtrsim 100 M_G$ when $h_t(M_G) \lesssim 1.5$. At large $\tan\beta$, GUT-scale uncertainties are under control, so that the measurement of SUSY parameters will be enough to make our analysis very accurate.

7. In summary, we have studied the impact on the low energy value of m_b/m_τ of an intermediate mass right-handed neutrino in a class of $SO(10)$ models. The analysis generally results in a lower bound on the mass M_N of the right handed neutrino, and thus in an upper bound on the τ neutrino mass and on its contribution Ω_ν to the hot dark matter density in the present universe. In order to do so we have performed a two-loop study of the RGE and discussed threshold effects at the GUT and SUSY scales. Depending on the values of α_s and $\tan\beta$ the bound varies from a very strong one to a weaker one.

i) When $\tan\beta \gtrsim 10$, there are uncertainties in the bound on M_N from the yet-unknown non-logarithmic SUSY threshold corrections. When these effects are maximal, then M_N is allowed to span all the phenomenologically interesting region (down to $M_N \sim 10^{12}$ GeV). On the other hand, when these effects are small, then only for very large $\tan\beta \gtrsim 50$ can we reach the cosmologically interesting region $M_N \lesssim 10^{13}$ GeV. We stress that effects at the SUSY-scale threshold will be known once SUSY particles are discovered and their parameters measured, so that such effects will be no longer ambiguities. Instead, we will be able to evaluate the corrections, which might even make the constraint stronger.

ii) For small $\tan\beta$, we get rather stringent bounds on Ω_ν. For $\alpha_s(m_Z) = 0.11, 0.117, 0.125$ we respectively get $\Omega_\nu h^2 < 4\times10^{-3}, 3\times10^{-5}, 6\times10^{-6}$ for $\tan\beta \leq 10$. Corrections from SUSY thresholds do not modify these bounds. If we allow for "maximal" GUT threshold effects of $\mathcal{O}(5\%)$, these bounds can go up by about two orders of magnitude. We point out that there might be additional corrections to $h_b/h_\tau = 1$ at M_G from physics beyond M_G. Assuming that the new physics is responsible for the flavor structure, we estimate the size of the corrections to be $\mathcal{O}(V_{cb}) \sim 5\%$. Even taking all these possible ambiguities into account, the cosmologically interesting region $\Omega_\nu \gtrsim 0.1$ is allowed typically only for $\alpha_s(m_Z) \lesssim 0.12$. Improvement in the experimental knowledge of α_s [39] will allow tighter bounds, especially if the value will converge towards the present central one $\gtrsim 0.12$.

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Note Added

After completing the main part of the work described in this letter and presenting it at a conference [40], we received a preprint by Vissani and Smirnov discussing a similar topic [41]. Their basic conclusion is that an intermediate mass right-handed neutrino disfavors b–τ Yukawa unification, or *viceversa*, which is the same as ours. However when deriving quantitative lower bounds on M_N, their analysis is based on one-loop RGE with tree-level matching, while ours is based on two-loop RGE with one-loop matching. We consider this to be necessary due to the high sensitivity of the bounds on M_N to $\mathcal{O}(5\text{–}10\%)$ effects which might arise from SUSY-scale and GUT-scale threshold corrections. In addition, they do not discuss the possible relevance of points (1) and (2) we discussed above. However, in the cases where all such effects are negligible, our result is consistent with theirs.

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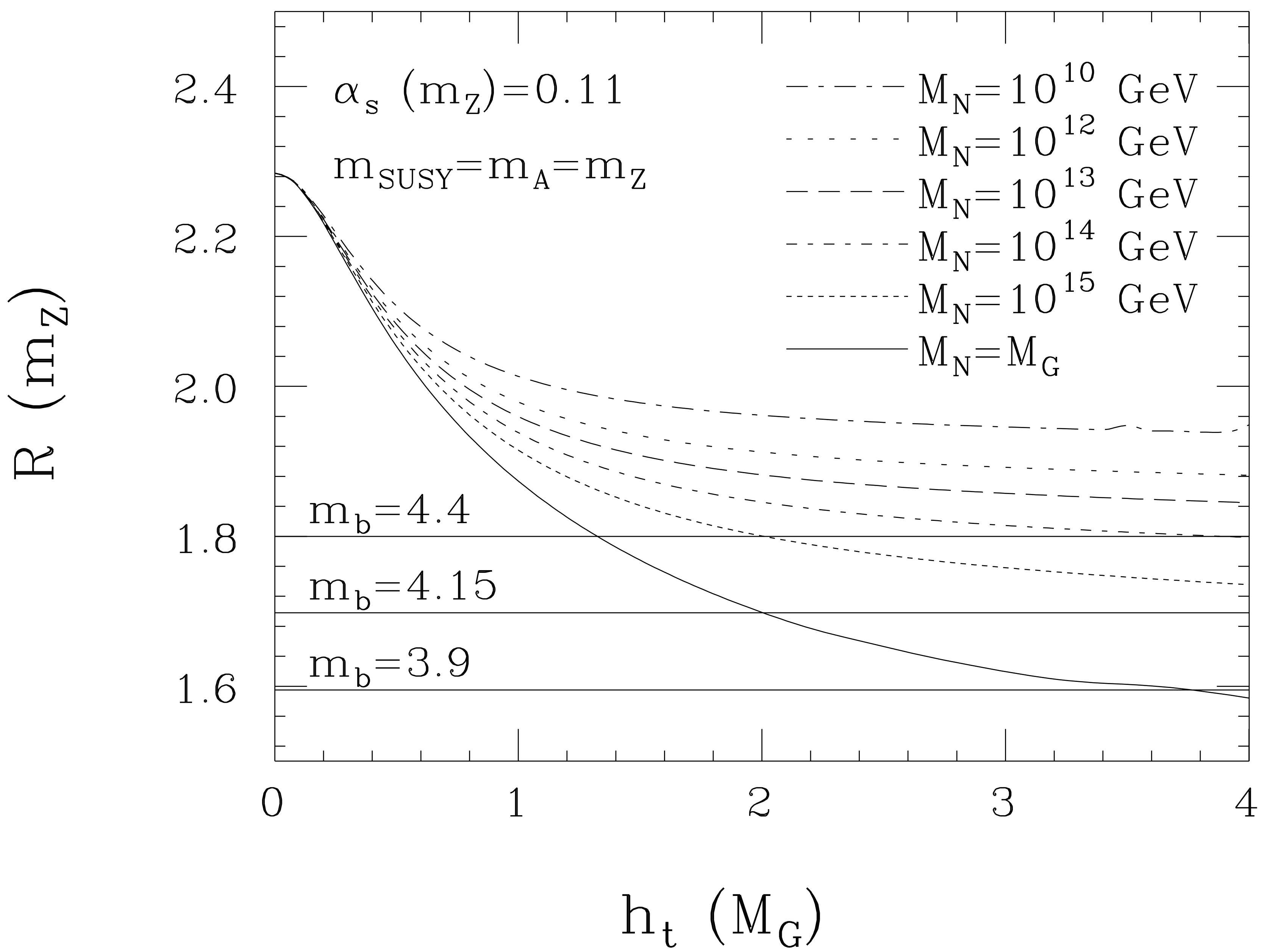
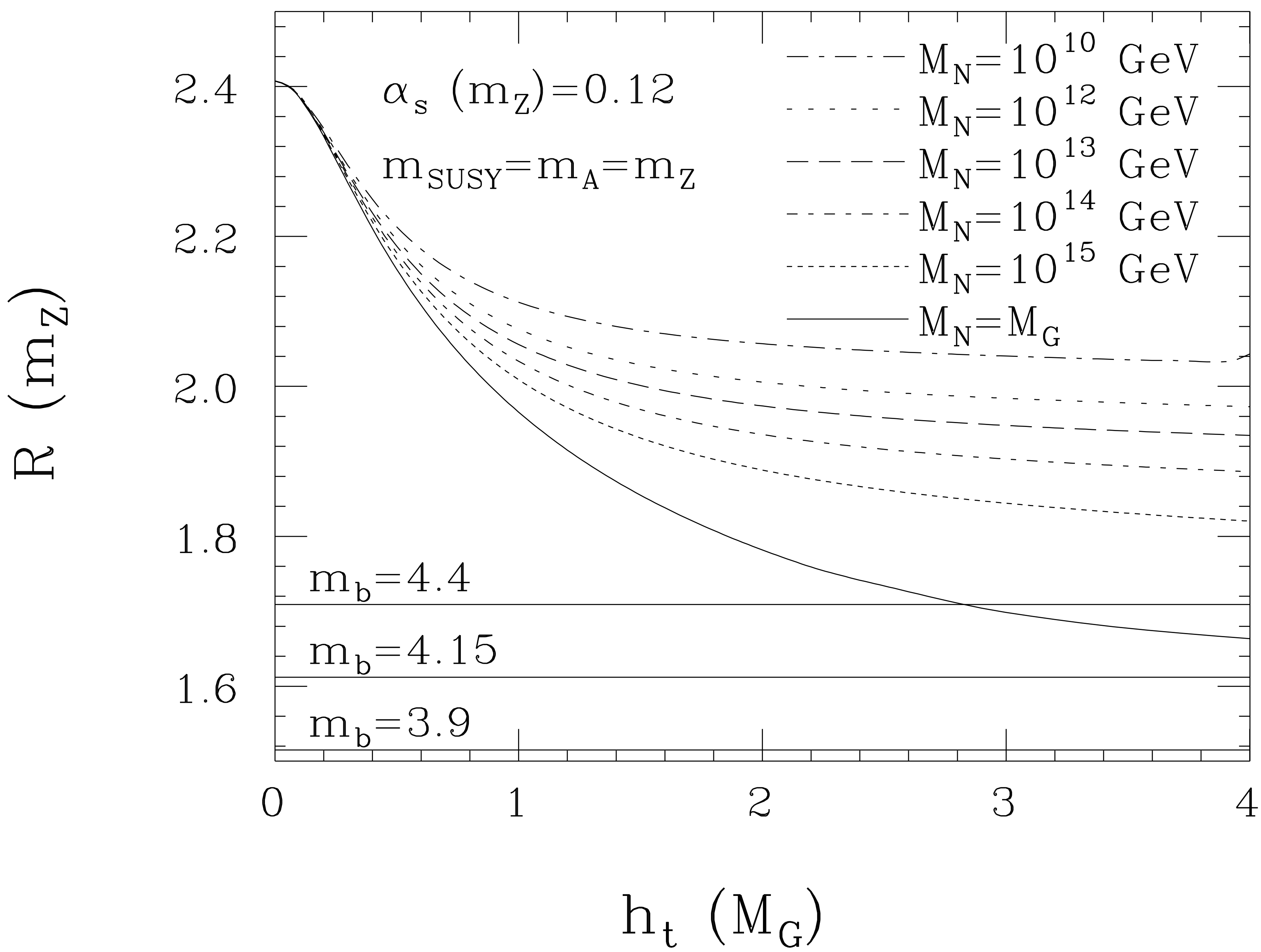
Figure Captions

Fig. 1 An illustration of the effect of the right-handed neutrino on $R(m_Z) = m_b(m_Z)/m_\tau(m_Z)$. The curves show the dependence of $R(m_Z)$ on $h_t(M_G) = h_N(M_G)$, for the values of $M_N = M_G$, 10^{15} GeV, 10^{14} GeV, 10^{13} GeV, 10^{12} GeV, and 10^{10} GeV. The experimentally allowed values of $R(m_Z)$ are also shown. Other input parameters are taken as $m_{SUSY} = m_A = m_Z$, $h_b(M_G) = h_\tau(M_G) = 0.01$, and $\alpha_s(m_Z) = 0.11$ (a) or 0.12 (b).

Fig. 2 Lower bounds on the right-handed neutrino mass M_N from b–τ Yukawa unification. Curves for three values of $\alpha_s = 0.110$ (solid), 0.117 (dots) and 0.125 (dash) are shown, both for the SUSY particle spectra (a) $m_{SUSY} = 1$ TeV, $m_A = m_Z$ (lower) and (b) $m_{SUSY} = m_A = m_Z$ (upper); see text. The possible cosmic energy density $\Omega_\nu h^2$ of the τ-neutrino hot dark matter is also shown for comparison in dot-dashed lines.



R (m_Z)
h_t (M_G)
α_s (m_Z)=0.11
m_SUSY=m_A=m_Z
M_N=10^10 GeV
M_N=10^12 GeV
M_N=10^13 GeV
M_N=10^14 GeV
M_N=10^15 GeV
M_N=M_G
m_b=4.4
m_b=4.15
m_b=3.9
2.4
2.2
2.0
1.8
1.6
0
1
2
3
4



R (m_Z)
h_t (M_G)
α_s (m_Z)=0.12
m_SUSY=m_A=m_Z
M_N=10^10 GeV
M_N=10^12 GeV
M_N=10^13 GeV
M_N=10^14 GeV
M_N=10^15 GeV
M_N=M_G
m_b=4.4
m_b=4.15
m_b=3.9
2.4
2.2
2.0
1.8
1.6
0
1
2
3
4

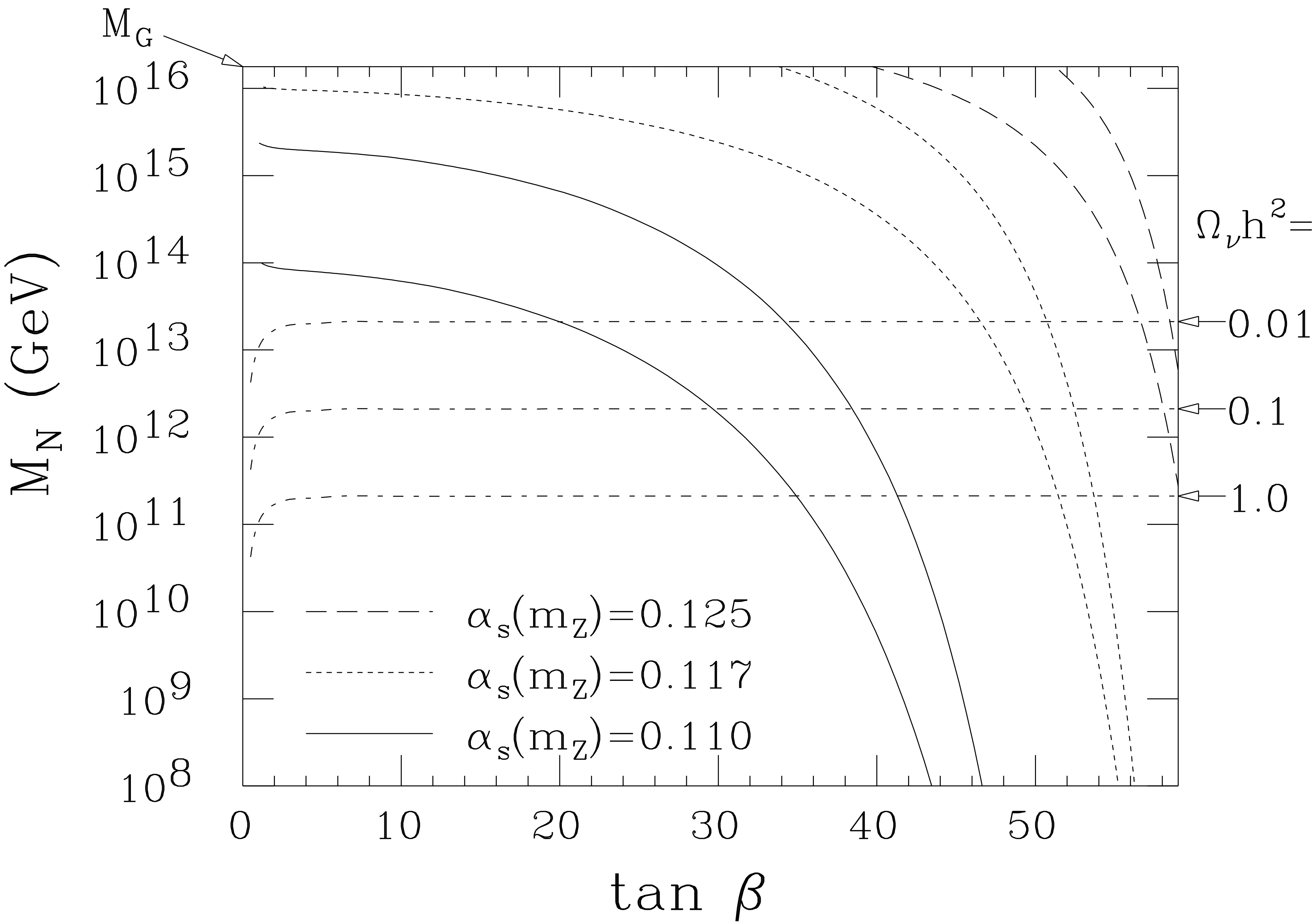

M_G
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
M_N (GeV)
0
10
20
30
40
50
tan β
$\Omega_\nu h^2=$
0.01
0.1
1.0
$\alpha_s(m_Z)=0.125$
$\alpha_s(m_Z)=0.117$
$\alpha_s(m_Z)=0.110$

Fig. 2